

Describing shape changes in the human mandible using elliptical Fourier functions

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SUMMARY Elliptical Fourier functions (EFF) were generated for the boundary outline of the mandibular form in 24 non-orthodontic growth study subjects at 9, 11, 13, and 15 years of age. The mean residual fit (observed versus predicted points) of the mandibular form was 0.11 mm, with a range from 0.09 to 0.14 mm, suggesting an accurate fit. The mean plot for each of the four age groups was area standardized and superimposed on the centroid. Shape changes in males and females were found to be similar. No significant shape changes were found between 9 and 11 years. Shape changes were observed in the lower incisor area, the mental region, the body of the mandible, and the gonial angle area between 11 and 15 years. These changes coincided with the onset of the pubertal growth spurt.

Introduction

Form can be considered as an aspect of fundamental importance in morphological investigations. Studies as diverse as the classification of species, diagnosis in pathology as well as identifying aircraft for military purposes, are based, one way or another, on the analysis of form. All biological forms consist of a large number of shared aspects that include size, shape, colour, structure, patterning, etc. However, the numerical characterization of such forms has been more of a challenge than may be at first realized. This is especially the case in morphological studies of biological organisms, which tend to be irregular in form. Moreover, even such concepts as size, shape, and form have not been without controversy. For example, form and shape have been used interchangeably. According to Webster (1983), form is defined as ‘the shape or outline of anything; figure; image; structure, excluding colour, texture and density’. Upon looking up ‘shape’, things become more confusing with shape defined as ‘... outline or external surface’ or ‘the form characteristic of a particular person or thing’. According to these definitions, ‘shape’

and ‘form’ are interchangeable, to be viewed as identical. Clearly, this unsatisfactory state of affairs is in need of re-definition. To alleviate some of these problems, a simple linear formulation has been proposed (Penrose, 1954) as:

$$\text{Form} = \text{Size} + \text{Shape}. \quad [1]$$

Size can be defined as a quantity that depends upon dimensional space. In a one-dimensional world, difference in size can be viewed as a difference in vector length. In two dimensions, linear measurements in combination (such as ratios) have proved to be inadequate, and area becomes one definition of size. The perimeter has also been utilized, although more influenced by boundary considerations (van Otterloo, 1991). In three dimensions, volume would be the appropriate size quantity. Shape, on the contrary, is a quantity that is difficult to adequately define, but has been characterized as ‘residual’, or what is left after controlling for size. A more technical definition for shape has been proposed as ‘that which remains invariant under scaling, translation, rotation and reflection’ (Lele, 1991). While we accept the latter definition, we also

stress the visual aspect, which leads us to heuristically define shape in terms of the boundary of the form.

One can control or standardize for size by making the bounded area equal for all comparisons. This procedure effectively diminishes the influence of size close to zero. Then, and only then, one can equate 'form with shape' and re-write equation 1 above to:

$$\text{Form} = \text{Shape}. \quad [2]$$

Thus, only when conditions are specifically defined can one begin to legitimately equate 'form' with 'shape'. To do otherwise only engenders confusion.

Since the discovery of X-rays and the invention of the cephalostat, a more meaningful comparison between individuals has become possible. The shape of the mandible is often used by clinicians as an aid in predicting mandibular growth and growth potential. The mandible is known to grow rapidly during the period of the pubertal growth spurt. This is generally agreed to occur between 11 and 15 years of age (Tanner, 1962). Whereas many studies have demonstrated the changes in size that may occur at this time, shape changes have been less well investigated. This is, in part, due to the lack of a user-friendly method of satisfactorily demonstrating the changes that occur.

Some of the current analytical methods used to analyse the biological form can be placed into two broad categories, which are: (1) homologous point and (2) boundary outline methods. These two approaches can be further divided into techniques that can be applied to data sets (Lestrel, 1989a, 1997a,b). Homologous point approaches include: (1) Conventional metrical analysis (CMA); (2) Biorthogonal grids (BOG); (3) Finite element method (FEM), related to BOG; (4) Euclidean distance matrix analysis (EDMA); and (5) Thin plate splines (TPS). Subsumed under boundary outline methods are: (1) medial axes analysis (MAA); (2) the resistant-fit theta rho analysis; (3) eigen shape analysis; and (4) elliptical Fourier functions (EFF). These are, by no means, the only methods, but represent those which to date have received the most attention. Each of these methods will be briefly outlined in

the following two sections. The interested reader is encouraged to peruse the cited references.

Homologous point methods

The CMA was originally developed for measuring regular geometric objects and its application to irregular biological forms may not be the most appropriate, in spite of its ease of use. This method consists of linear measurements of distances, angles, and ratios. Its application to biological forms for describing size and shape, except in the simplest cases, is generally unsatisfactory because: (1) a limited number of isolated landmarks are usually employed; (2) there is an unavoidable bias in the choice of landmarks; and (3) difficulties in adequately standardizing for size (Lestrel, 1982, 1989a). Thus, much of the information that resides in biological forms, so readily apparent in visual terms, is not being adequately measured. These and other factors stimulated the development of alternative methods to circumvent the deficiencies of CMA.

BOG and FEM are related methods. The FEM has been borrowed from engineering where it is a well-established procedure for measuring the effect of stresses or loadings on engineering materials. While the method of BOG represents the simplest form of FEM, they are both dependent on homologous points or landmarks, which have to be identifiable across the forms that are to be compared (a presumed advantage of these methods is that they are both independent of the co-ordinate system).

In BOG, any three homologous points can form a reference triangle (Bookstein, 1977, 1978). A concentric circle can then be drawn touching all three sides of the triangle. With growth or treatment, changes in the position of the homologous point means a new triangle is formed. If the relative position of the circle's contact points is maintained, the deformation of the reference triangle into the second would transform the circle into an ellipse. Thus, the circle would be effectively stretched along one axis and the direction of this distortion may be calculated. The longest and shortest diameters of the ellipse are known as its principal axes, and these lengths can be measured. The greater the discrepancy between them, the

larger the difference in shape. Bookstein (1977, 1978) suggested that the change in shape be expressed as a ratio, the larger axis measurement divided by the smaller. Changes in size were described as the product of the two lengths. This method measures size and shape changes between two forms, rather than size or shape of a single form. That is, it is not a description of the form, as it would be visualized.

In FEM, the triangles in BOG are replaced by hexahedrons or cubes. Each cube element is composed of eight homologous points in the x , y , z Cartesian plane. Using an initial form as a base, each element in the structure is pairwise compared and the shape change is computed as a deformation. As before, these cube elements are independent of each with the exception of where the surfaces are joined. These elements are 'non-homogeneous' in the sense that they represent spatially varying tensor fields, in contrast to BOG in which the principal dilations are based on a constant tensor field. For each element, it is possible to estimate a 'form difference' tensor that is equal to a 'shape difference' tensor plus a 'size difference' tensor. Shape changes are then found by subtraction. By averaging these shape changes at all nodes and over the whole form, summary estimates can be obtained.

The FEM can be extended to three dimensions, an advantage over the BOG. In addition, the cube elements are 'non-homogeneous' in that they represent spatially varying tensor fields in contrast to BOG in which the principal dilations are based on a constant tensor field. Once again, the FEM deals with changes in size and shape, rather than actual numerical descriptions of size or shape.

Euclidean distance matrix analysis (EDMA) was largely developed for applications in biology. It is also a landmark-dependent method that is applicable to two- and three-dimensional structures (Lele, 1991; Lele and Richtsmeier, 1991). It was developed to avoid some of the problems inherent in the superimposition of landmark-based outlines. This method, as with CMA, FEM and BOG, is a co-ordinate-free method. EDMA separately calculates all possible Euclidean distances between landmarks for each morphology under consideration. This generates a distances

matrix called a 'Form matrix' (Lele and Richtsmeier, 1991). Pairwise comparisons are then made between morphological cases by computing a matrix of ratios of the corresponding linear distances. This leads to a Form difference matrix. From these ratios, it is possible to identify what areas of the morphology exhibit the largest shape differences.

Finally, a comparatively new technique is thin plate splines (TPS). Using the theory of surface spline interpolations (Bookstein, 1991), the development of TPS is a continuation of Bookstein's work on deformations started with BOG. This represents an approach to put Thompson's (1915, 1942) Cartesian transformation grids on a more solid mathematical footing. However, the method of BOG did not provide for a sophisticated graphical display of the point-to-point deformations sought by Bookstein (Reyment, 1991). The following discussion of TPS is intended to briefly convey this method.

TPS is a technique for visualizing form change as a deformation. It uses an interpolation function representing a mapping that models the 'biological homology' of pairs of points. The interpolant can be thought of as a smooth (well-behaved) function that is fitted to a data point set. In two dimensions these would be the familiar spline functions such as the Bezier curves. The TPS function may be visualized as an infinitely thin metal plate placed over a set of landmarks. This surface allows visualization the pairwise displacement of landmarks as a deformation. This is carried out by computing the 'bending energy'. If this plate is 'flat' then it has zero bending energy. If bending energy is involved, it acts to 'wrinkle' the plate in some fashion. The larger the deformation, the greater the bending energy and the more 'buckled' the thin plate becomes. These TPS deformations can also be expressed as a sum of the 'principal warps'. These are eigen vectors of the bending energy which correspond to the orthogonal displacements of the landmarks (above and below) from the thin plate [for further exposition of this method refer to Bookstein (1991) and Reyment (1991)].

The three deformation methods: BOG, FEM, and TPS, as well as EDMA, can all be considered

as novel contributions to morphometrics. They represent sophisticated mathematical methods for eliciting new information present in the biological form. They are also attempts to deal with the shortcomings inherent with CMA. These methods, however, also share one aspect with CMA, which is the dependency on homologous points. Nevertheless, because of this landmark-based constraint, none of these approaches can be considered as complete models of form in themselves as one aspect remains missing, which is the boundary or curve information.

Boundary outline methods

Medial axis analysis (MAA), is based on the symmetric axis of an outline form passing down a precisely defined middle of an extended structure (Blum, 1973; Lavelle, 1985). The medial axis of a curve is simply the collection of all the points that are 'in the middle'. That is, a curve can be constructed by connecting the centres of circles that touch the boundary of the form at two distinct points. This curve, plus an expression of its distance from the boundary (based on the radii of the circles), is sufficient to completely describe the shape of the structure. These medial axes provide 'stick' figures for complex biological forms, which serve as a means for registering slow changes in curvature, relative position of parts, etc.

The resistant-fit theta rho analysis has not received much attention (Benson *et al.*, 1982; Siegel and Benson, 1982). The method is useful for morphologies where it is known that the variability of substructures is not randomly distributed across the form. The advantage of this method is that it is relatively stable against departures from the assumptions of analysis, such as independent, identically, and normally distributed errors. It is also more stable against the potentially strong influences of atypical or incorrect data values. For instance, two shapes can rarely be superimposed perfectly; different fitting criteria will generally yield different results. By allowing regions with large deformations to have a large impact on the fit, methods such as the least squares methods can potentially minimize true shape differences and thereby

obscure them. A resistant technique, however, limits the influence of large deformations and the resulting fit is close in similar regions and not so close in relatively deformed regions. In this way, resistant techniques can help to identify similarities and differences in form more effectively than least squares methods. Siegel and Benson (1982) have demonstrated the superiority of the resistant fitting technique over the least squares fit technique.

Eigen shape analysis represents another approach for numerically describing the outline and thereby the shape of organisms. This technique was developed using data (comparatively smooth microfossil outlines) derived from palaeontology. It is based on the presumption that the use of eigen shape analysis facilitates the reduction of the morphological shape space to a comparatively few dimensions (Lohmann, 1983; Schweitzer *et al.*, 1986). Eigen shape analysis is misleading in one sense—it actually represents the sequential use of two techniques, rather than representing a single development. Eigen shape analysis is the initial application of Zahn and Roskies' (1972) Fourier-based algorithm, the results of which are then subsequently subjected to a factor analytical method. Lohmann and Schweitzer (1990) proposed three measures for describing planar outlines within the context of eigen shape analysis. These were form, size and angularity. Form referred to two aspects of the outline: scaling (size) and amplitude (variance). Form was computed using the standardized (for size and variance) formulation of Zahn and Roskies' Fourier formulation. Size was defined two ways, as the perimeter of the outline and by the bounded area. Angularity was measured by the magnitude of the amplitudes of the angular part of Zahn and Roskies' algorithm, but before these amplitudes were standardized to unit variance.

Zahn and Roskies' formulation has the advantage that it is always a single-valued function and does not require a vector centre such as the centroid. Lohmann (1983) correctly indicated that if the centroid was not used, incorrect computed amplitudes were the result. This had been noted earlier by Parnell and Lestrel (1977), and Full and Ehrlich (1982).

However, the use of n equal-divisions as advocated by Lohmann (1983), negates the possibility of maintaining homology. As Full and Ehrlich (1986) have argued, there is very little possibility that homology between outlines can be consistently maintained with eigen shape analysis. Finally, a comparison of a number of Fourier methods (Rohlf and Archie, 1984), suggested that Zahn and Roskies' algorithm was the least satisfactory (Rohlf, 1986). Thus, morphologies with complex irregularities such as the mandible cannot easily be handled with either conventional Fourier methods or eigen shape analysis.

Elliptical Fourier functions (EFF) represent a significant step forward in the quest for a method that efficiently captures boundary outline information (Kuhl and Giardina, 1975). One of the early criticisms of conventional Fourier analysis was that homology of points across forms was lost. With EFFs, this problem has now been rectified. Homology is now maintained by a specific computational procedure: the first homologous predicted point is computed to be at the same location on the Fourier approximating (interpolating) function as the first digitized point is on the digitized form (observed data file). The second and subsequent homologous predicted points are computed so that they have the same arc length from the first computed point as their counterpart (pseudo-) landmarks from the first digitized point on the original digitized curve. This maps the pseudo-landmark points from the digitized curve onto the EFF (Wolfe, 1997). This is equivalent to shifting the observed co-ordinates of the polygonal representation of the form, onto the EFF curve (Lestrel and Huggare, 1997). This shift is quite small since it is incumbent upon the investigator to keep the residual, the difference between the observed points and the predicted points derived from the EFF, as small as practically possible. Mean values based on all points should not rise above 0.10–0.20 mm, i.e. these values need to be well below the errors arising from: (1) locating the points, (2) tracing the cephalograms, and (3) digitizing. The EFF software now maintains the homology of the points, and hence, maintains the homology of the entire mandibular form. The authors term these points pseudo-homologous,

after the suggestions of Sneath and Sokal (1973). The application of this technique to cephalometry has been described in some detail by Lestrel (1997b), and Lestrel and Kerr (1993).

The present study is associated with the use of EFF in analysing the shape changes in the human mandible. Since the majority of orthodontic treatment is carried out in growing children, analysis of shape changes in the mandible during this period can be obscured by the concurrent change in its size due to active growth. One particularly useful characteristic of the EFF is its ability to control for or standardize the size of the mandible while maintaining its shape. Thus, the aims of the present study were to: (1) identify the differences, if any, in the shape of the mandible between the sexes; and (2) localize the areas of shape changes in the mandible during the period of the pubertal growth spurt.

Materials and methods

Cephalograms of subjects taken at 9, 11, 13, and 15 years of age were selected from Leighton's archive growth study material (Bhatia and Leighton, 1993). The age bands were selected to maximize the sample size and observe the changes that take place around puberty. None of the subjects had undergone orthodontic treatment. Cephalograms showing more than 5–6 degrees of rotation at the mandibular border were rejected. A total of 11 female and 13 male subjects satisfied the above criteria representing 96 cephalograms in total. The mean age of the subjects at each age point was 113.6, 137.8, 162.5, and 185.6 months for the females, and 113.2, 137.1, 163.1, and 186.1 months for the males.

The mandibular outline was traced on each cephalogram with a Staedtler 6H pencil onto 0.003-inch, non-stretched matt acetate. Seventy-eight points (Figure 1) were added to the outline with a Rotring Rapidograph roller ball-point pen with a 0.18-mm nib. A full description of the tracing protocol and points employed has been reported (Lestrel and Kerr, 1993). A back-lit GTCO 15 digitizer (SSI Microcad, Pewsey, Wilts, UK) and an IBM PC AT computer were used to run the specially developed EFF software package (EFFA; Lestrel, 1982, 1989a,b, 1997b).

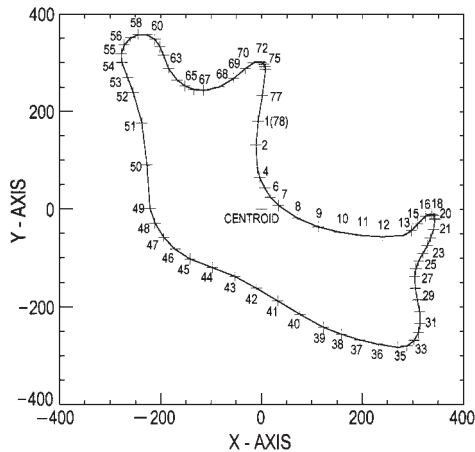


Figure 1 Mandibular point system.

Statistical analyses were carried out using Minitab for Windows version 9.0 and the descriptive statistics package in the EFFA.

Data handling according to the EFFA protocol

The tracings were orientated with point 1 (on the anterior surface of the vertical ramus) coincident with the centre of the digitizer screen and the maxillary plane coincident with the horizontal axis of the digitizer. Each tracing was thus digitized, the resultant mandibular outline being referred to as an *observed form*. The data from the observed form was then used to compute the *predicted form* using a stepwise procedure based on harmonic coefficients. A maximum number of 39 harmonics were computed to achieve the best possible fit. The maximum number of harmonics is subject to Nyquist frequency constraints which dictate that the number of harmonics be one-half the number of observed data points (Lestrel, 1997b). A residual value was then calculated in millimetres. This residual was the difference between the observed data points and the predicted values derived from the EFF. These residual values were averaged over the whole form. A maximum residual value of 0.5 mm was tolerated. For anatomical work residual values of less than 0.3 mm can be considered good. As previously shown by Lestrel

and Kerr (1993) the elliptical Fourier fit to a mandibular form is a stepwise procedure. The first harmonic represents an ellipse. With the addition of 10 harmonics, the residual was 0.76 mm overall. With 39 harmonics the mean residual dropped to less than 0.11 mm.

Each predicted mandibular form was then area standardized to 10,000 mm² in order to eliminate the size factor while retaining the shape of the form. Mean plots of the area-standardized mandibular form were subsequently calculated for each age and sex group by averaging the predicted plots, referenced to each case's centroid. These mean plots were superimposed for graphical comparison. The distances from the centroid to the predicted landmark points on the boundary were also calculated from the area-standardized form. This vector of distances summarized the available information about shape.

Method error

Five subjects were selected at random and their radiographs (20), re-traced and re-digitized. The centroid position on the first tracing was then used to calculate the centroid to boundary points distances on the two tracings. Student's *t*-test, *t*-interval, and intra-class correlation between the boundary points of the first and second tracing were computed using the above data.

Although a *P*-value of <0.05 is generally considered significant with the Student's *t*-test, for the purposes of this study, a Bonferroni correction was made. This was required because 18 homologous points were assessed in one test, so that some of the points were bound to be statistically significant at the 5 per cent level by chance alone. The Bonferroni correction makes allowance for this by requiring a *P*-value of <0.003 to be significant at the 5 per cent level.

Only the 18 *key* points out of 78 were considered in assessing the measurement error as the rest of the points were dependent on the accuracy of these key points. Key points represent, for the most part, traditional anatomical landmarks used in cephalometrics. With regard to the *t*-test, only point 13 (just lingual to the lower incisor) had a *P*-value of <0.003, a statistically significant error. Perhaps with erupting teeth in

that area, the level of alveolar bone was difficult to determine. The t -interval showed that points 13, 72 (the most superior aspect of the coronoid process), and 78 (where ANS–PNS line bisects the anterior ramus) did not contain zero. Thus, significant changes relating to these points should be interpreted with caution.

The correlation of points between the first and second tracing showed the majority of the key points had correlation coefficients of >0.6 , with the exception of points 62, 67, 72, and 76. The authors considered the key points at the sigmoid notch and coronoid process the most difficult to locate, with respect to the film quality presented, and the anatomically overlapping shadows of the zygoma and the maxilla.

Analysis of the data

Hotelling's T^2 test on the 18-dimensional vector of centroid-to-key-point distances was carried out to investigate the differences in shape between male and female subjects at each of the four ages in turn. It was found that there were no significant gender differences between the male and female groups allowing the data to be combined. The combined data now consisted of 24 subjects in each age group.

Interest was then focused on the presence of age differences. The mean centroid-to-boundary distances for each mandibular area-standardized age group were then calculated and the statistical differences between each group assessed and summarized (Table 1). The graphic presentations of the results, superimposed on the centroids, are shown in Figures 2 and 3.

The main changes seen in the various regions of the mandible were as follows:

The lower incisor. There was no observable change in the angulation of the lower incisor between 9 and 11 years of age. The lingual movement of the lower incisor from age 11 to 15 was obvious.

Point B. Point B followed the same trend as the lower incisor.

Pogonion. It seems that pogonion is relatively stationary.

Table 1 Significant differences (*), $P < 0.05$, with Bonferroni correction, between age groups.

	Age (years)					
	9–11	11–13	13–15	9–13	11–15	9–15
Point						
1	–	–	–	–	–	–
3	*	–	–	*	–	*
13	–	–	*	–	*	–
18	–	*	*	–	*	–
23	–	*	*	–	*	*
27	–	*	*	–	*	*
31	–	–	*	–	*	–
35	–	–	–	–	–	–
39	–	–	*	*	–	–
43	–	*	*	–	*	–
49	–	–	–	–	*	–
51	–	–	–	–	–	*
54	–	–	–	–	–	–
58	–	–	–	–	–	–
62	–	–	–	–	–	–
67	–	–	–	–	*	–
72	–	–	–	–	*	–
76	–	–	–	–	*	*
78	–	–	–	–	–	–

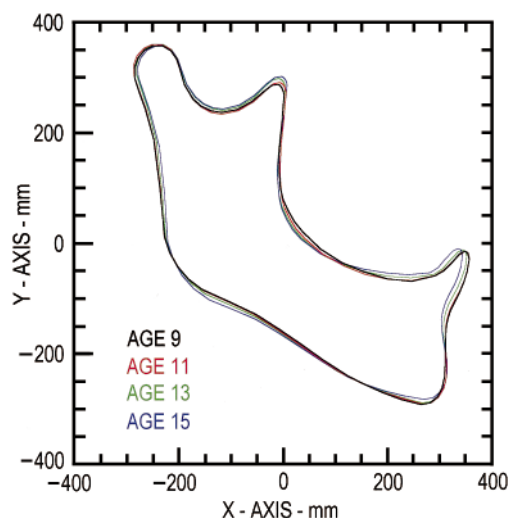


Figure 2 Mandibular shape changes.

The chin. The prominence of the chin was derived from relative lingual movement of the lower incisor and point B.

The mandibular body. The anterior portion of the body seemed to have bone deposition in the

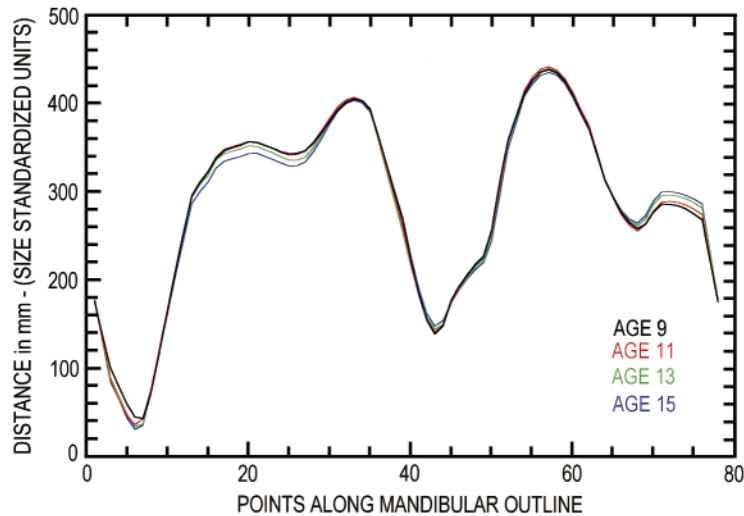


Figure 3 Centroid to boundary distances.

upper border and remodelling in the lower border. The posterior portion, however, showed a completely opposite course. The net effect appeared to be one of anterior rotation.

The mandibular ramus. The condylar shape remained more or less constant. The coronoid process showed a steady increase in convexity. The gonial angle also reduced as growth progressed.

There was no shape difference between 9 and 11 years of age. Between 11 and 13 years of age, points 18, 23, 27 and 43 showed significant changes. Between 13 and 15 years of age, points 13, 18, 23, 27, 31, 39, and 43 showed significant changes. Points 1, 35, 54, 58, 62, and 78 remained stationary throughout growth. Points 49, 51, 72, and 76 demonstrated no significant change in position between consecutive films, but showed cumulative significant differences.

Discussion

EFF and traditional cephalometric analysis

Classical cephalometric analysis depends upon the identification of anatomical landmarks on lateral cephalograms and the construction of lines and planes to form a reference system. EFF

is slightly less landmark-dependent, but still requires a point (point 1) and a line (maxillary plane) for initial orientation purposes. The computer then generates the centre of the area enclosed—the centroid. Using this centroid a series of distances are computed to the mandibular boundary. One essential feature, however, is that the effect of size can be excluded, leaving only shape differences to be analysed.

Traditionally, Fourier coefficients and/or their amplitudes have been used to numerically describe the biological form. Such measurements allow for a global analysis of form (Lestrel, 1982, 1989a). However, a drawback is that it is difficult to relate specific values of the Fourier coefficients to the actual shape of the form under investigation. The present study used a set of centroid-based distances, which allow for a localized analysis of specific regions. Nevertheless, it must be re-iterated that all shape changes reported are with respect to the centroid.

EFF as a curve fitting tool based on residuals

The average residual value after 39 harmonics was 0.11 mm. Points 18 (tip of the incisor) and 72 (the tip of the coronoid process) were those displaying the poorest fit. This is explained by

the fact that the change in contour is particularly abrupt at these anatomical aspects. Even so, the goodness of fit had only diverged by an average 0.57 and 0.34 mm for these two points, respectively. These residuals can be considered to be well within the error margin of the method. Due to the closeness of the overall EFF fit, all the points on the whole outline can be analysed, rather than just the limited information provided by the location of a few selected landmark points.

Sample mix, size, and timing

The sample consisted of mixed skeletal morphology, the purpose being to investigate the average shape change in a population, rather than those of individual cases. An approximately equal mix of female and male subjects was considered acceptable. However, the subject number was small (24). This was limited first by the number available within the archive and, secondly, the age group required in this study. The age group was chosen to encompass the pre-pubertal, pubertal spurt, and post-pubertal periods. However, the information available within the archive was such that it was not possible to determine the peak height velocity for each subject.

The centroid and superimposition

Johnston (1960) first advocated the use of centroids for the analysis of cephalograms. The centroids chosen were those of the cranium and the face. The cranio-facial-centroid or the CFC line was found to be less variable than any of the traditional cephalogram axes by a circular variances study (Johnston and Laycock, 1980). Other cephalometric studies, which used the centroid for their orientation, include Johnston (1978), Wastall *et al.* (1988), and Trenouth (1987, 1993). The allometric-centred model of cranio-facial growth, which was so strongly advanced by Moss *et al.* (1983), however, is quite a different concept. A centre of allometry is defined as a small neighbourhood of 'nearly zero'. Thus, its location is much more imprecise than the mathematical centroid. A practical drawback of

the centroid approach, however, is the fact that it makes comparison with traditional distance measurements more difficult.

A number of other planes have been used for superimposition and orientation. Nanda (1955) stated that there is no landmark in the human head that is truly stationary. Therefore, there is no ideal location for superimposition purposes. Broadbent (1937) and Brodie (1940) suggested that the cranial base remains stable throughout growth. Ford (1958) later demonstrated that the sella and nasion both migrate superiorly, and there is no evidence that the cranial base is of the same shape or lies in the same orientation in the skulls of different individuals.

Cook and Southall (1989) assessed three methods of superimposition in the mandible:

1. The mandibular plane, being defined as a line drawn between the cephalometric point menton and gonion;
2. The mandibular outline;
3. Björk's mandibular structures (Björk, 1969) as:
 - (a) the anterior contour of the bony chin;
 - (b) the inner contour of the cortical plate at the lower border of the symphysis;
 - (c) the contour of the mandibular canal;
 - (d) the lower contour of the mineralized lower third molar tooth germ prior to root formation.

They found sizeable errors associated with all three groups, but tracings involving Björk's mandibular structures were found to be the least reliable. However, they did recommend the use of Björk's structures when the time interval between radiographs was small. Miller and Kerr (1992) used four different planes for superimposition. The sella-nasion plane holding at Sella, Menton-point 1 at menton, maxillary plane holding at anterior nasal spine, and the Xi-Pm point at Pm. A greater appreciation of growth was possible using various reference planes. Thus, innovative approaches such as the EFF used in this study, with the use of a calculated centroid for data analysis, should be welcome.

Changes shown in this investigation compared with other studies

So far, only two studies of a similar nature have been carried out (Halazonetis *et al.*, 1991; Ferrario *et al.*, 1996). In the former study, a conventional Fourier analysis approach was used. However, only the lower border of the mandible from gnathion to articulare was investigated. The reason being that no abrupt changes in curvature occurred. The outline also approximates to a circular arc and, hence, it was easier to use polar co-ordinates in the EFF from the centre of this arc, thus reducing the number of Fourier coefficients. The sample size was larger than the present study with 55 female and 39 male subjects. Three films, 2 years pre-pubertal, pubertal, and 2 years post-pubertal were used. Using the Fourier coefficients for their analysis, they found a decrease in the gonial angle with age. This is in agreement with the present study. The study of Ferrario *et al.* (1996) used the 18-year-old sex averaged, Bolton study templates, and a semi-automatic outline digitizing technique. The Bolton material used to produce the templates was part longitudinal and part cross-sectional. Whilst the bulk of that study was concerned with change in size, the shape aspect of the study was mainly confined to changes between three time points, at age 1, 6, and 18 years of age. They also found minimal shape changes after 15 years. Consequently it is difficult to make direct comparisons with the results of this study.

Opinions on the shape of the mandible and sexual dimorphism often lack consensus. Sex differences in mandibular shape were not found in this investigation. Halazonetis *et al.* (1991) found sex-related differences in shape in the ages studied with male subjects showing a more rounded mandible than females. Nevertheless, statistical differences were small. The gonial angle, however, did not differ in size between the sexes. Gilmore (1950), Ingerslev and Solow (1975), and Bibby (1979), also found no differences in the pattern of craniofacial morphology in the male and female mandible, contrary to Jensen and Palling (1954), and Horowitz and Thompson (1964). It is difficult, however, to

satisfactorily describe shape differences using conventional linear and angular measurement on cephalometric radiographs.

Nevertheless, many of the findings of this investigation are in agreement with traditional views, e.g. the lingual movement of the lower incisor and point B, and the formation of the chin. The relative stationary nature of pogonion (point 35) and anterior rotation of the mandible was also demonstrated. However, the increase in convexity of the coronoid process should be viewed with caution as point 72 also has a sizeable error. The fact that not all points behave similarly between consecutive radiographs implies that, even within the mandible, the rate of shape change varies with age, as well as with the location. Significant shape changes start to occur from 11 years onwards.

Conclusions

1. The satisfactory nature of EFF as a curve-fitting tool in biological forms is demonstrated.
2. Mean shape changes in the mandible showed no differences between sexes in the age range studied.
3. There were no appreciable shape differences between the mean 9- and 11-year-old mandibles.
4. The rate of mandibular shape changes varied among different morphological aspects of the mandible.
5. The lower incisor and point B moved relatively lingually with growth; hence, the chin became more developed.
6. Shape changes in the mandibular body contributed to the anterior rotation of the mandible.
7. The expected increase in convexity of the coronoid process was small, but demonstrable on aggregates, although this result should be viewed with caution.

Future work on EFFs could involve the use of long- or short-face type subjects. Different skeletal class samples could also be analysed separately. Work is currently underway to streamline the EFFA software so that all procedures, calculating coefficients, normalization for size, computation

of centroid-based distances, etc., are automated. A module has been recently added that facilitates the measurement of specific distances between points.

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